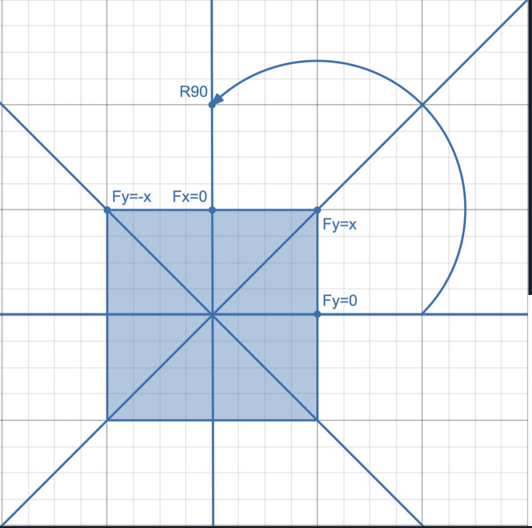


The symmetries of the square, $D_4 = \{ R_0, R_{90}, R_{180}, R_{270}, F_{y=0}, F_{y=x}, F_{x=0}, F_{y=-x} \}$, is generated by R_{90} and $F_{y=0}$.

g ->	R0	R90	R180	R270	Fy=0	Fx=0	Fy=x	Fy=-x
R90 o g	R90	R180	R270	R0	Fy=x	Fy=-x	Fx=0	Fy=0
Fy=0 o g	Fy=0	Fy=-x	Fx=0	Fy=x	R0	R180	R270	R90

Task 1:

- 8 people should represent the 8 elements of D_4
- Each person holds a marker. Together, find an arrangement in which you can accomplish these two passing actions.
    - Red Pass: $g \longrightarrow R_{90} \circ g$
    - Blue Pass: $g \longrightarrow F_{y=0} \circ g$
    - The goal is for all 8 people to pass at the same time.

Groups en Action

Brian P Katz (BK), CSULB
Jessi Lajos, USU
MathFest’24, Indianapolis, Inquiry TCPS

Abstract

Inspired by prior work of Jessi Lajos and team, I wanted to incorporate some embodied cognition in my abstract algebra course to help my students make meaning of the most challenging ideas (and to have some fun). In this session, we will discuss an activity that spanned three class days in which my students acted out algebraic structures, including the properties of groups, generation, cosets, homomorphisms, and quotients. I hope to share the tasks, some student thinking, and my experience ramping up an embodied task in my classroom.

Impetus

(1) I have been frustrated in the past that I had not successfully helped students build a robust, meaningful understanding of Quotient Groups.

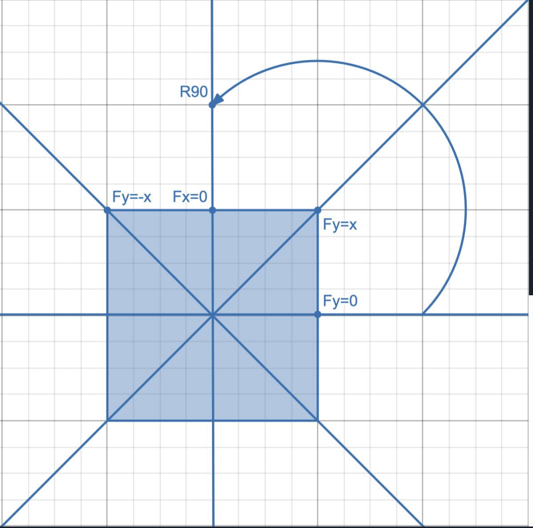
(2) I wanted my Abstract Algebra course to model diverse kinds of active pedagogies because all of my students are future high school teachers.

(3) Jessi Lajos's dissertation work in embodiment focused on embodying homomorphisms, which seemed like a great launching point for (1) and (2).

(4) There were still challenges in the embodied context of Jessi's work.

Plan: Teach a multi-day unit via embodiment in my course!

Topics: Cosets, normality, quotient groups, homomorphisms, first isomorphism theorem, more!



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R90 o g	R90	R180	R270	R0	Fy=x	Fy=-x	Fx=0	Fy=0
Fy=0 o g	Fy=0	Fy=-x	Fx=0	Fy=x	R0	R180	R270	R90

Task 1:

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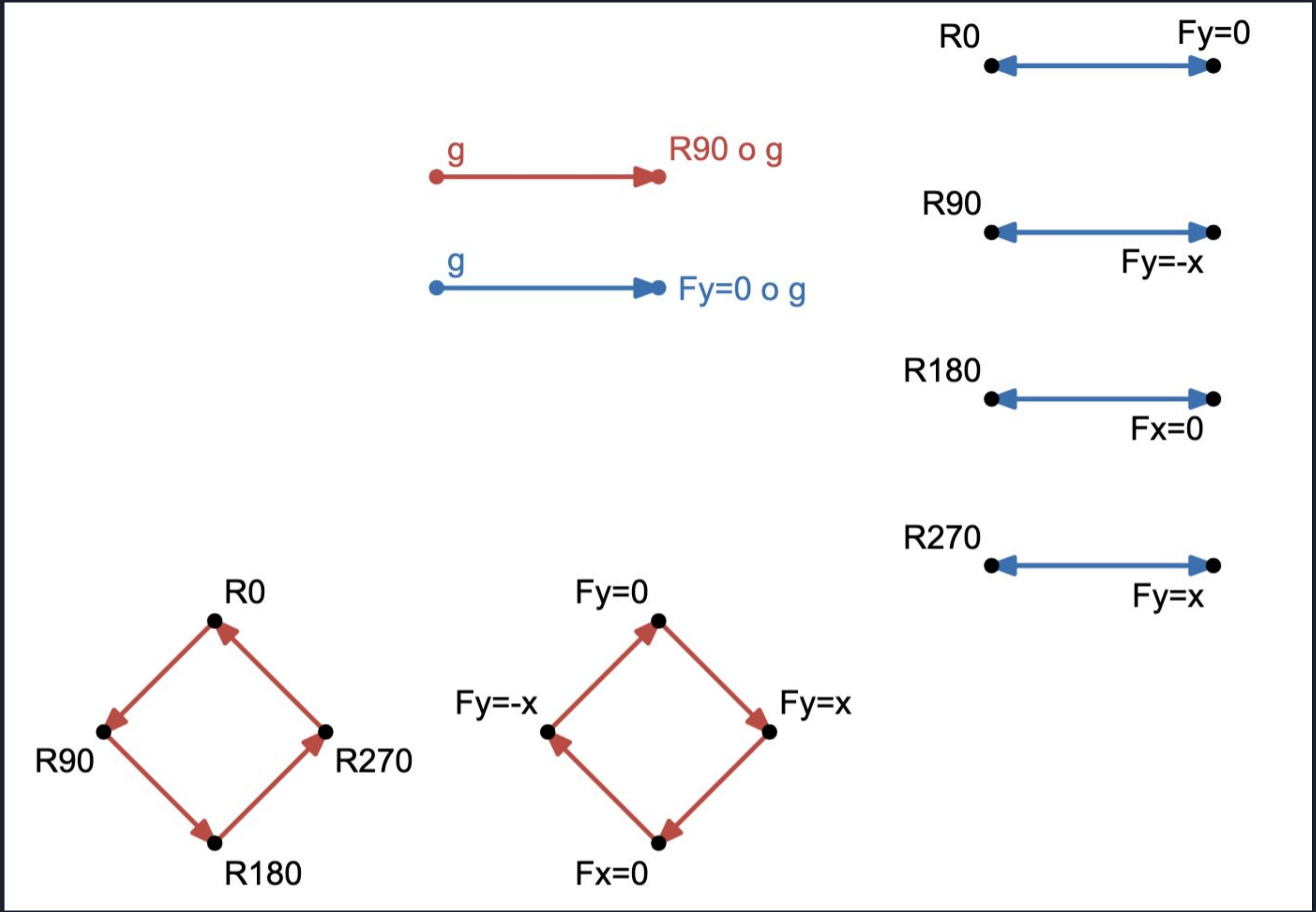
R0
Fy=0
g
R90 o g
R90
Fy=-x
g
Fy=0 o g
R180
Fx=0
R270
Fy=x
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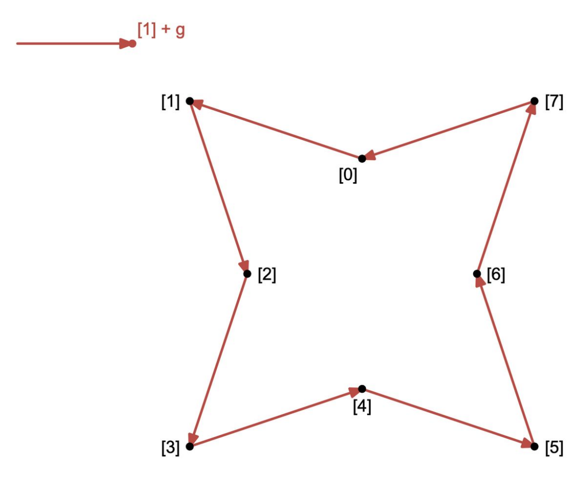
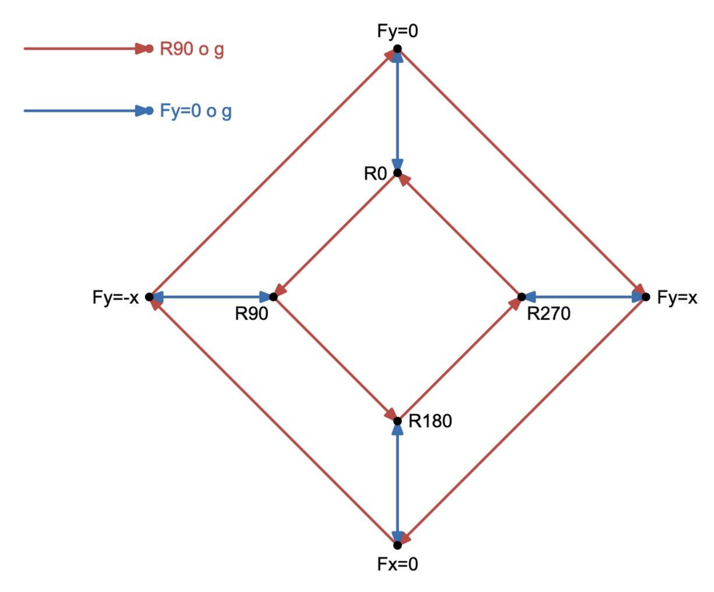
R90
R270
R180
Fy=0
Fy=-x
Fy=x
Fx=0

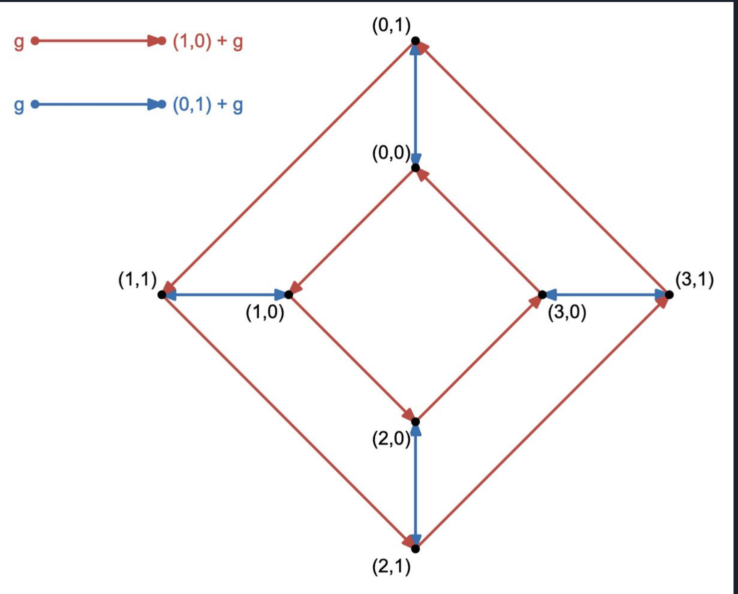
$\mathbb{Z}_8$

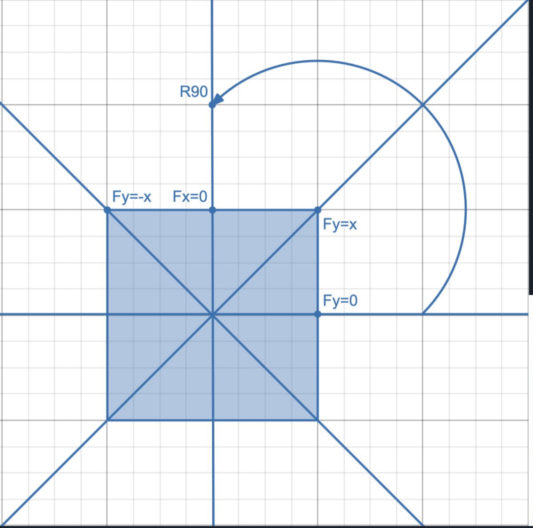


D_4



$\mathbb{Z}_4 x \mathbb{Z}_2$





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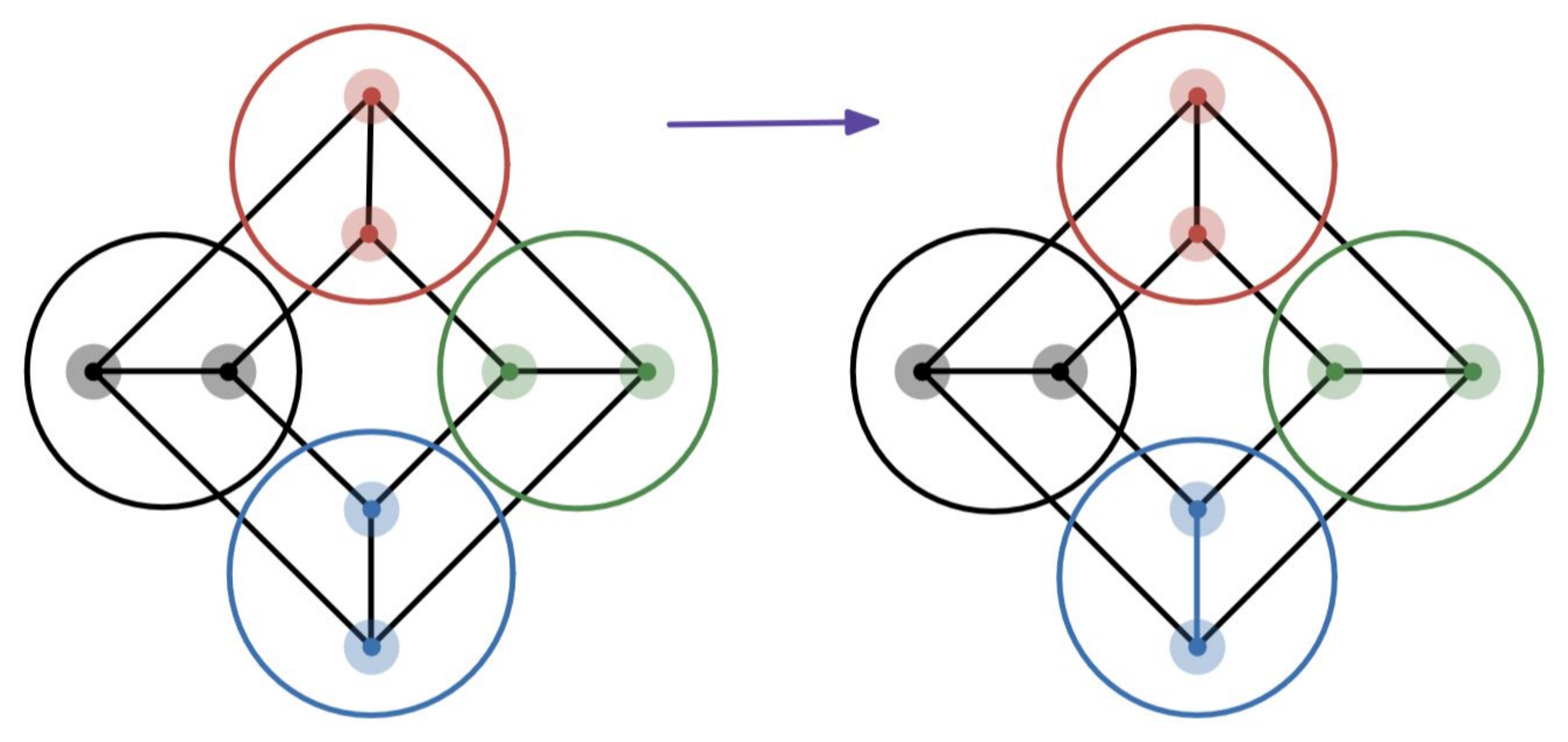
R90 o g	R90	R180	R270	R0	Fy=x	Fy=-x	Fx=0	Fy=0
Fy=0 o g	Fy=0	Fy=-x	Fx=0	Fy=x	R0	R180	R270	R90

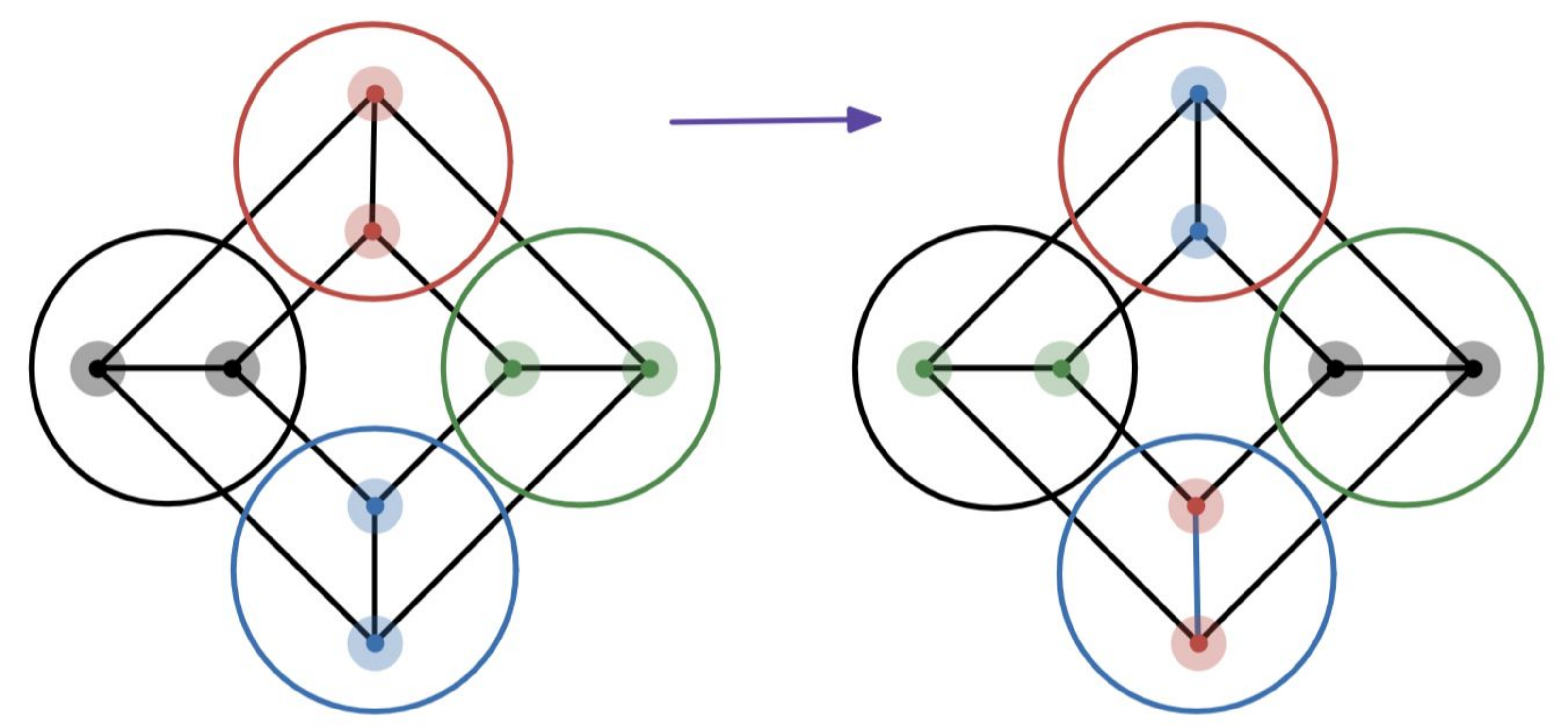
Task 2:

- 8 people should represent the 8 elements of D4
- Each with has a necklace: 2 red, 2 green, 2 blue, 2 black
- Each person holds a marker with the color of their necklace. Do the colors of the markers/necklaces, "play nicely" with the two passing actions?
  - Red Pass: $g \longrightarrow R_{90} \circ g$
  - Blue Pass: $g \longrightarrow F_{y=0} \circ g$

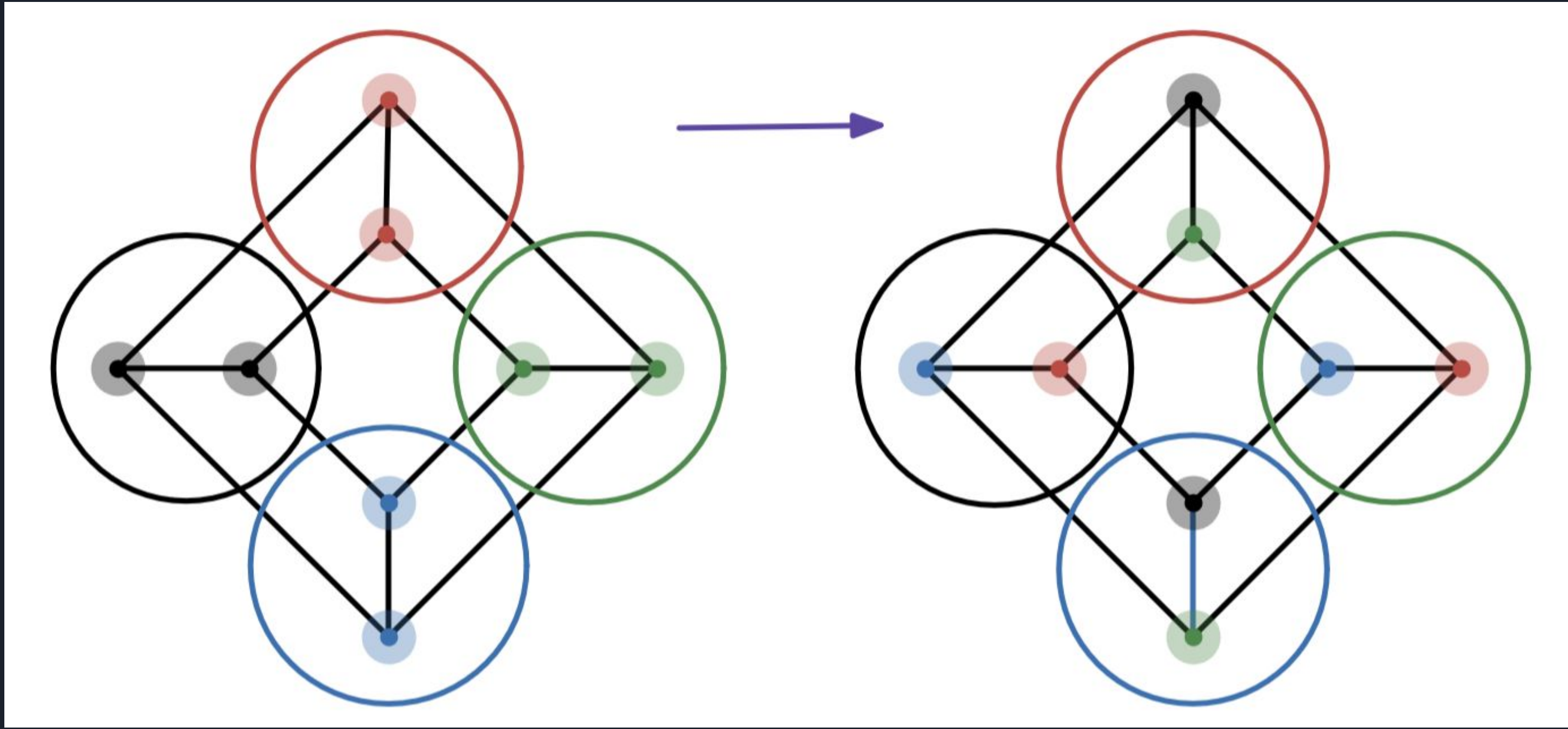
Blue pass ($F_{y=0}$):



Double red pass (R_{180}):



Red pass (R_{90}):



Task Phases: Day 1

Embodied Generation

- Embody the 8 elements of D_4, $\mathbb{Z}_4 x \mathbb{Z}_2$, and $\mathbb{Z}_8$ (via generators)
- Enact operations of generators
- Explore closure, well-definedness, Cancellation, operation/composition, generation, commutativity
- Partition the group with colors
- Do the generators and partition play nicely together?
- Explore subgroup, and maybe cosets/Lagrange's Theorem

Task Phases: Day 2

Embodied Operation

- Embody the 8 elements of D_4, $\mathbb{Z}_4 x \mathbb{Z}_2$, and $\mathbb{Z}_8$
- Enact operations of arbitrary pairs of elements
- Explore operation/composition, associativity, identity, inverses, commutativity [closure, well-definedness, Cancellation]
- Partition the group with colors
- Do the operations and partition play nicely together?
- Explore Lagrange's Theorem, subgroup, and maybe cosets

[*This is the revised activity, improving on what I actually did by changing the order.]

	O	A	B	C	D	P	Q	R	S	T	U	V	W	X	Y	Z
O	O	A	B	C	D	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	O	P	Q	R	B	C	D	V	W	X	S	T	U	Z	Y
B	B	P	O	S	T	A	V	W	C	D	Y	Q	R	Z	U	X
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D	D	R	T	U	O	W	X	A	Y	B	C	Z	P	Q	S	V
P	P	B	A	V	W	O	S	T	Q	R	Z	C	D	Y	X	U
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Z	Z	Y	X	W	V	U	T	S	R	Q	P	D	C	B	A	O

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Z	O	Q	T	Z	T	Z	O	Q	Q	O	Z	T	Z	T	Q	O

Task Phases: Day 3

Compressing Partitions into Quotients

- Assess which trial partitions play nicely
- Explore what is required for a partition to play nicely
- Explore subgroup, cosets, normal, subring, ideal/strong closure, Lagrange's Theorem
- Explore quotients to see how all other properties of a group/ring are induced

[*This is the revised activity, improving on what I actually did by changing the order.]

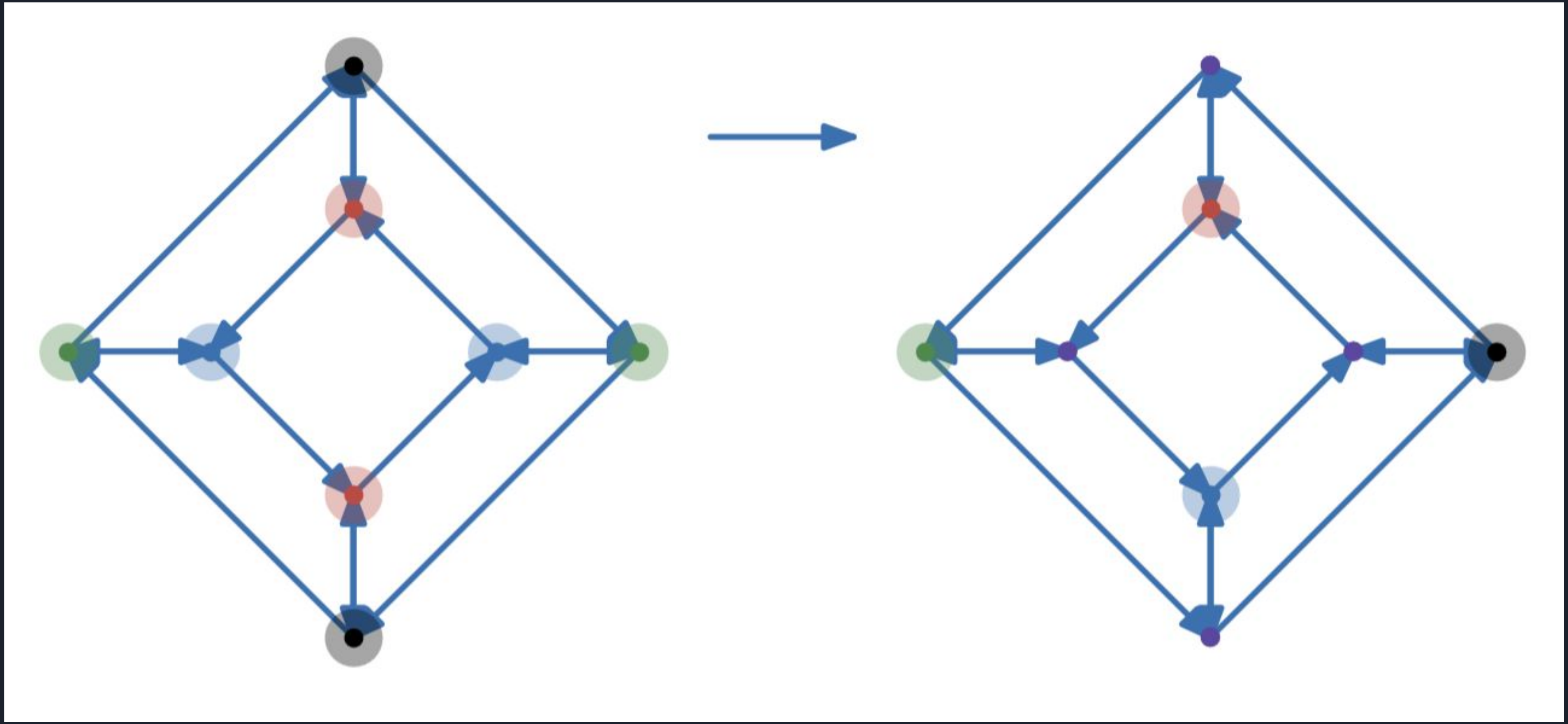
Task Phases: Day 4

Embodying Homomorphisms and the First Isomorphism Theorem

- Embody homomorphisms
- Assess which candidate mappings are homomorphism
- Partition the domain with colors and align with the codomain
- Explore subgroup, cosets, fibers, kernel, image/preimage sets and subgroups, other theorems about homomorphisms
- Explore the image as a quotient of the domain mod kernel (First Isomorphism Theorem)

[*This is the revised activity, improving on what I actually did by changing the order.]

$$D_4 / \{R_0, R_{180}\} \cong \{ (0,0), (2,0), (0,1), (2,1) \} \subseteq Z_4 x Z_2$$



Impetus

(1) I have been frustrated in the past that I had not successfully helped students build a robust, meaningful understanding of Quotient Groups.
(2) I wanted my Abstract Algebra course to model diverse kinds of active pedagogies because all of my students are future high school teachers.
(3) Jessi Lajos's dissertation work in embodiment focused on embodying homomorphisms, which seemed like a great launching point for (1) and (2).
(4) There were still challenges in the embodied context of Jessi's work.

Plan: Teach a multi-day unit via embodiment in my course!
Topics: Cosets, normality, quotient groups, homomorphisms, first isomorphism theorem, more!

Thanks/Questions!

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Jessi: jessi.lajos@usu.edu

Pedagogy and research papers coming!